

Neutrino clustering in the galaxy with a global monopole

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In spherically symmetric, static spacetime, we show that only $j = 1/2$ fermions can satisfy both Einstein's field equation and Dirac's equation. It is also shown that neutrinos are able to have effective masses and cluster in the galactic halo when they are coupled to a global monopole situated at the galactic core. Astronomical implications of the results are discussed.

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I. INTRODUCTION

In Barriola and Vilenkin's global monopole solution [1] to Einstein's field equation, scalar fields with global $O(3)$ broken symmetry are minimally coupled to gravity and the background spacetime has deficits of angle. Nucamendi *et al.* [2] suggested that the global monopole solution could explain the flat rotational velocity curves (FRVC) of stars in galaxies because its energy density is proportional to $1/r^2$ and hence it can be dark matter in the galactic halo. Even if some questions are issued [3] about the global monopole solution for FRVC, various generalized versions of the global monopole were studied as models for dark matter and dark energy [4].

Neutrino clustering was studied by some authors including one of us [5] to explain the continuation of the cosmic ray spectrum beyond the GZK (Greisen, Zatsepin and Kuzmin) cutoff. Neutrino-antineutrino ($\nu\bar{\nu}$) annihilation to a Z^0 -boson is one possible explanation for the phenomena. But for $\nu\bar{\nu}$ annihilation into a Z^0 -boson to produce super GZK air shower, a neutrino flux at $E_\nu \geq 10^{21} \text{ eV}$ with $m_\nu \sim \text{a few eV}$ and a significant clustering of the relic neutrino density in our galactic halo are required [6].

In this article we investigate if neutrinos can coalesce into neutrino clouds in a curved spacetime. Considering fermions coupled to an $O(3)$ triplet of scalar fields in the most general static metric with spherical symmetry, we show that only the total angular momentum $j = 1/2$ fermions can satisfy both Einstein's field equation and Dirac's equation. When the global $O(3)$ symmetry of the Lagrangian is spontaneously broken to $U(1)$ at the ground state of the scalar potential in the shape of Mexican hat, we can have a global monopole solution similar to that of Barriola and Vilenkin. Being coupled to a global monopole situated at the galactic center, neutrinos are able to have effective masses and cluster in the galactic halo.

II. GLOBAL $O(3)$ SYMMETRIC MODEL FOR NEUTRINO CLUSTERING

The action of the global $O(3)$ symmetric model for scalar fields Φ^m and massive neutrinos Ψ_n ($m, n = 1, 2, 3$) minimally coupled to gravity is given by

$$S = \int d^4x \sqrt{-g} \mathcal{L}, \quad (1)$$

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi^m \partial_\nu \Phi^m - V(\vec{\Phi}^2) \\ & + \frac{i}{2} (\bar{\Psi}_n \gamma^a \nabla_a \Psi_n - \nabla_a \bar{\Psi}_n \gamma^a \Psi_n) - m_n \bar{\Psi}_n \Psi_n \\ & + \frac{g_y}{v} \Phi^m \Phi^m \bar{\Psi}_n \Psi_n, \end{aligned} \quad (2)$$

where $V(\vec{\Phi}^2)$ is a scalar potential and the last term is the scalar-neutrino interaction for which we assume the first leading order term preserving $O(3)$ symmetry, with the symmetry breaking scale v of the more fundamental theory. Varying the action with respect to the fields, we obtain following equations for scalar fields Φ^m and neutrinos Ψ_n :

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi^m) - \frac{\partial V}{\partial \Phi^m} + 2 \frac{g_y}{v} \bar{\Psi}_n \Psi_n \Phi^m = 0, \quad (3)$$

$$i \gamma^a \nabla_a \Psi_n - m_n \Psi_n + \frac{g_y}{v} \vec{\Phi}^2 \Psi_n = 0, \quad (4)$$

where the γ^a -matrices satisfy the Clifford algebra in a locally flat inertial coordinate;

$$\{\gamma^a, \gamma^b\} = -2\eta^{ab} \quad (5)$$

with

$$\eta^{ab} = \text{Diag}(-1, 1, 1, 1), \quad (6)$$

and the covariant derivative

$$\nabla_a = e^\mu_a (\partial_\mu + \Gamma_\mu) \quad (7)$$

is constructed from the vierbein e^μ_a and spin connection Γ_μ , which we give explicitly in Eqs. (25)-(31).

Using the standard definition of the energy-momentum tensor

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = -\frac{e_{a\mu}}{\det\{e\}} \frac{\delta S}{\delta e^\nu_a}, \quad (8)$$

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we have

$$T_{\mu\nu} = \partial_\mu \Phi^m \partial_\nu \Phi^m - g_{\mu\nu} \left[\frac{1}{2} \partial^\beta \Phi^m \partial_\beta \Phi^m + V(\vec{\Phi}^2) \right] - \frac{i}{4} [(\bar{\Psi}_n \gamma_\mu \nabla_\nu \psi_n - \nabla_\nu \bar{\Psi}_n \gamma_\mu \Psi_n) + (\mu \leftrightarrow \nu)]. \quad (9)$$

The energy-momentum tensor allows us to construct the Einstein equation,

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \quad (10)$$

where $G_{\mu\nu}$ is the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \quad (11)$$

with Ricci tensor $R_{\mu\nu}$ and $\kappa = 8\pi G$.

When we consider the following potential of scalar fields

$$V(\vec{\Phi}^2) = \frac{\lambda}{4} (\vec{\Phi}^2 - \eta^2)^2, \quad (12)$$

with a constant η , the global $O(3)$ symmetry is spontaneously broken to $U(1)$. The scalar field configuration describing a global monopole is known to be given by the hedgehog ansatz,

$$\Phi^m = F(r) \frac{x_m}{r}, \quad (13)$$

with a real function $F(r)$ and $r = (x_m x_m)^{1/2} = (x^2 + y^2 + z^2)^{1/2}$,

Assume the line element of the spherically symmetric, static spacetime as

$$ds^2 = -\delta^2(r) \alpha^2(r) dt^2 + \frac{dr^2}{\alpha^2(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (14)$$

In the Cartesian coordinates,

$$x^\mu = (t, x^m) = (t, x, y, z), \quad \text{for } m = 1, 2, 3, \quad (15)$$

the line element can be written by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad (16)$$

with the metric tensor

$$g_{\mu\nu} = -\delta^2 \alpha^2 \delta^\mu_t \delta^\nu_t + [\delta_{mn} + (\frac{1}{\alpha^2} - 1) \frac{x_m x_n}{r^2}] \delta^\mu_m \delta^\nu_n \quad (17)$$

and its inverse

$$g^{\mu\nu} = -\frac{1}{\delta^2 \alpha^2} \delta^\mu_t \delta^\nu_t + [\delta_{mn} + (\alpha^2 - 1) \frac{x_m x_n}{r^2}] \delta^\mu_m \delta^\nu_n. \quad (18)$$

From the standard definition of Christoffel symbols

$$\Gamma_{\alpha\beta}^\mu = \frac{1}{2} g^{\mu\nu} (g_{\nu\alpha, \beta} + g_{\nu\beta, \alpha} - g_{\alpha\beta, \nu}), \quad (19)$$

we derive

$$\begin{aligned} \Gamma_{tm}^t &= \frac{x_m}{2r} \left(\frac{\delta^2_{,r}}{\delta^2} + \frac{\alpha^2_{,r}}{\alpha^2} \right), \\ \Gamma_{tt}^l &= \frac{x_l \alpha^2}{2r} (\delta^2 \alpha^2_{,r} + \delta^2_{,r} \alpha^2), \\ \Gamma_{mn}^l &= \frac{x_l \alpha^2}{2r} \left[-\frac{\alpha^2_{,r}}{\alpha^4} \frac{x_m x_n}{r^2} + (\alpha^{-2} - 1) (\delta_{mn} - \frac{x_m x_n}{r^2}) \frac{2}{r} \right], \\ \text{Others} &= 0. \end{aligned} \quad (20)$$

In the metric given in Eq. (14), scalar field equation reads

$$\begin{aligned} \alpha^2 F_{,rr} + \frac{\alpha^2}{2} F_{,r} \left(\frac{\delta^2_{,r}}{\delta^2} + 2 \frac{\alpha^2_{,r}}{\alpha^2} + \frac{4}{r} \right) - \frac{2F}{r^2} \\ - \frac{\partial V}{\partial F} + \frac{2g_y}{v} \bar{\Psi}_n \Psi_n F = 0, \end{aligned} \quad (21)$$

where $F_{,r} \equiv \frac{\partial F}{\partial r}$, $F_{,rr} \equiv \frac{\partial^2 F}{\partial r^2}$,

We rewrite the line element as

$$ds^2 = \eta_{ab} e^a e^b \quad (22)$$

with inverse of η^{ab} , $\eta_{ab} = \text{Diag}(-1, 1, 1, 1)$ and

$$e^0 = \alpha \delta dt, \quad e^1 = \frac{dr}{\alpha}, \quad e^2 = r d\theta, \quad e^3 = r \sin \theta d\phi. \quad (23)$$

Putting

$$e^a = e^a_\mu dx^\mu, \quad (24)$$

we find the vierbein given by

$$e^a_\mu = \begin{pmatrix} \alpha \delta & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha} \sin \theta \cos \phi & \frac{1}{\alpha} \sin \theta \sin \phi & \frac{1}{\alpha} \cos \theta \\ 0 & \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ 0 & -\sin \phi & \cos \phi & 0 \end{pmatrix} \quad (25)$$

and their inverses

$$e^\nu_b = \begin{pmatrix} \frac{1}{\alpha \delta} & 0 & 0 & 0 \\ 0 & \alpha \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ 0 & \alpha \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ 0 & \alpha \cos \theta & -\sin \theta & 0 \end{pmatrix} \quad (26)$$

$$= \begin{pmatrix} \frac{1}{\alpha \delta} & 0 & 0 & 0 \\ 0 & (\alpha \frac{\partial x^m}{\partial r}) & (\frac{1}{r} \frac{\partial x^m}{\partial \theta}) & (\frac{1}{r \sin \theta} \frac{\partial x^m}{\partial \phi}) \end{pmatrix}. \quad (27)$$

The vierbein satisfy the relations

$$e^a_\mu e^\mu_b = \delta^a_b, \quad (28)$$

$$e^\nu_a e^a_\mu = \delta^\nu_\mu. \quad (29)$$

The spin connection which appeared in Eq. (7) is defined as

$$\Gamma_\mu = -\frac{1}{4} \gamma^a \gamma^b e_{\nu a} e^\nu_{b;\mu} \quad (30)$$

with

$$e^\nu{}_{b;\alpha} = e^\nu{}_{b,\alpha} + \Gamma^\nu_{\alpha\beta} e^\beta{}_{b.} \quad (31)$$

We calculate the spin connections from the above Eq. (20) and Eq. (30), and obtain

$$\begin{aligned} \Gamma_t &= \frac{1}{4}\gamma^0\gamma^1(2\alpha^2\delta_{,r} + \delta\alpha^2{}_{,r}) \\ \Gamma_x &= \frac{1}{2}\gamma^1\gamma^2\frac{\alpha\cos\theta\cos\phi}{r} - \frac{1}{2}\gamma^2\gamma^3\frac{\cos\theta\sin\phi}{r\sin\theta} \\ &\quad + \frac{1}{2}\gamma^3\gamma^1\frac{\alpha\sin\phi}{r}, \\ \Gamma_y &= \frac{1}{2}\gamma^1\gamma^2\frac{\alpha\cos\theta\sin\phi}{r} + \frac{1}{2}\gamma^2\gamma^3\frac{\cos\theta\cos\phi}{r\sin\theta} \\ &\quad - \frac{1}{2}\gamma^3\gamma^1\frac{\alpha\cos\phi}{r}, \\ \Gamma_z &= -\frac{1}{2}\gamma^1\gamma^2\frac{\alpha\sin\theta}{r}, \end{aligned} \quad (32)$$

which satisfy the Cartan structure equation,

$$[\gamma^\nu, \Gamma_\mu] = \gamma^\nu{}_{;\mu}. \quad (33)$$

III. SPHERICAL SYMMETRY AND $j = \frac{1}{2}$ FERMIONS

We can show that neutrino field equations given by

$$i\gamma^\alpha e^\mu{}_\alpha(\partial_\mu + \Gamma_\mu)\Psi_n - m_n\Psi_n + \frac{g_y}{v}\vec{\Phi}^2\Psi_n = 0 \quad (34)$$

become

$$\begin{aligned} &\gamma^0\frac{i}{\alpha\delta}\partial_t\Psi_n + \gamma^1i\alpha\hat{D}_r\Psi_n + \gamma^2\frac{i}{r}(\partial_\theta + \frac{\cos\theta}{2\sin\theta})\Psi_n \\ &+ \gamma^3\frac{i}{r\sin\theta}\partial_\phi\Psi_n - (m_n - \frac{g_y}{v}\vec{\Phi}^2)\Psi_n = 0, \end{aligned} \quad (35)$$

in the spherically symmetric and static metric in Eq. (14). Equivalently, we can write the last equation as

$$i\partial_t\Psi_n = \hat{H}\Psi_n, \quad (36)$$

where the Hamiltonian is defined as

$$\hat{H} = \alpha\delta[\gamma^0\gamma^1\frac{\alpha}{i}\hat{D}_r + \gamma^1\frac{1}{ir}\hat{k} + \gamma^0(m_n - \frac{g_y}{v}\vec{\Phi}^2)], \quad (37)$$

with

$$\hat{D}_r = \partial_r + \frac{1}{r} + \frac{1}{4}(\frac{\delta^2{}_{,r}}{\delta^2} + \frac{\alpha^2{}_{,r}}{\alpha^2}). \quad (38)$$

Here we have defined the operator \hat{k} as [7] [8]

$$\hat{k} \equiv i\gamma^0\gamma^1(\gamma^2\frac{1}{i\sqrt{\sin\theta}}\partial_\theta\sqrt{\sin\theta} + \gamma^3\frac{1}{i\sin\theta}\partial_\phi), \quad (39)$$

which commutes with the Hamiltonian operator \hat{H}

$$[\hat{k}, \hat{H}] = 0. \quad (40)$$

Taking the representation of γ -matrices to be direct products of independent Pauli matrices σ_i, ρ_j ($\vec{\rho} \otimes \vec{\sigma}$ representation) [7] [8],

$$\gamma^0 = \rho_2, \gamma^1 = i\rho_1, \gamma^2 = -i\rho_3\sigma_3, \gamma^3 = -i\rho_3\sigma_1, \quad (41)$$

we can represent \hat{k} in 2-dimensional σ -space

$$\hat{k} = \sigma_3\frac{1}{i\sqrt{\sin\theta}}\partial_\theta\sqrt{\sin\theta} + \sigma_1\frac{1}{i\sin\theta}\partial_\phi, \quad (42)$$

and we can solve the eigenvalue equation,

$$\hat{k}\mathcal{Y}_k^m(\theta, \phi) = k\mathcal{Y}_k^m(\theta, \phi), \quad (43)$$

with 2-dimensional spinor spherical harmonics [8]

$$\begin{aligned} \mathcal{Y}_k^m(\theta, \phi) &= \frac{e^{im\phi}}{\sqrt{2\pi}} \left[\frac{(j+m)!}{(j-m)!} \right]^{1/2} \frac{(\tan\frac{\theta}{2})^{\sigma_2/2}}{\sin^m\theta} \times \\ &\left(\frac{\partial}{\partial\cos\theta} \right)^{j-m} \frac{\sin^2j\theta}{2^j(j-\frac{1}{2})!} (\tan\frac{\theta}{2})^{-\sigma_2} u_k, \end{aligned} \quad (44)$$

where

$$u_k = \frac{1}{\sqrt{2}|k|} \begin{pmatrix} |k| \\ k \end{pmatrix} \quad (45)$$

with $k = \pm(j + \frac{1}{2})$ and $j = l \pm \frac{1}{2}$. In the representation (41) for γ -matrices, the Hamiltonian is expressed in terms of 2-dimensional ρ -matrices

$$\hat{H} = \alpha\delta[\rho_3\frac{\alpha}{i}\hat{D}_r + \rho_1\frac{1}{r}\hat{k} + \rho_2(m_n - \frac{g_y}{v}\vec{\Phi}^2)] \quad (46)$$

The eigenfunctions of both \hat{H} and \hat{k} can be written as direct products of the ρ -space spinor $\psi_n^{(km)}$ and σ -space spinor \mathcal{Y}_k^m ,

$$\Psi_n(t, r, \theta, \phi) = e^{-iE_nt} \sum_{-j \leq m \leq j} \psi_n^{(km)}(r) \mathcal{Y}_k^m(\theta, \phi), \quad (47)$$

and the neutrino equations $i\partial_t\Psi_n = \hat{H}\Psi_n$ then read

$$\frac{E_n}{\alpha\delta}\psi_n^{(km)} = \rho_3\frac{\alpha}{i}\hat{D}_r\psi_n^{(km)} + [\rho_1\frac{k}{r} + \rho_2(m_n - \frac{g_y}{v}\vec{\Phi}^2)]\psi_n^{(km)}. \quad (48)$$

Moreover, if we put

$$\psi_n^{(km)}(r) \equiv \frac{1}{r}(\alpha^2\delta^2)^{\frac{-1}{4}} R_n^{(km)}(r), \quad (49)$$

since

$$\hat{D}_r\psi_n^{(km)} = \frac{1}{r}(\alpha^2\delta^2)^{\frac{-1}{4}} \partial_r R_n^{(km)}, \quad (50)$$

then we get

$$\frac{E_n}{\alpha\delta}R_n^{(km)} = \rho_3\frac{\alpha}{i}\partial_r R_n^{(km)} + [\rho_1\frac{k}{r} + \rho_2(m_n - \frac{g_y}{v}\vec{\Phi}^2)]R_n^{(km)}. \quad (51)$$

Setting

$$R_n^{(km)} = \begin{pmatrix} \varphi_n^{(km)} \\ \chi_n^{(km)} \end{pmatrix}, \quad (52)$$

we obtain the coupled equations,

$$\begin{aligned} \left(\frac{E_n}{\alpha\delta} + i\alpha\partial_r\right)\varphi_n^{(km)} &= \left[\frac{k}{r} - i(m_n - \frac{g_y}{v}\vec{\Phi}^2)\right]\chi_n^{(km)} \\ \left(\frac{E_n}{\alpha\delta} - i\alpha\partial_r\right)\chi_n^{(km)} &= \left[\frac{k}{r} + i(m_n - \frac{g_y}{v}\vec{\Phi}^2)\right]\varphi_n^{(km)}, \end{aligned} \quad (53)$$

asymptotic solutions to which are given in the following section.

Einstein's equation $G_{\mu\nu} = \kappa T_{\mu\nu}$ can be rewritten as

$$R_{\mu\nu} = \kappa(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T_\alpha{}^\alpha). \quad (54)$$

Substituting Eq. (9) into the Einstein's equation in Eq. (54) gives

$$\begin{aligned} R_{\mu\nu} = & \kappa[\partial_\mu\Phi^m\partial_\nu\Phi^m + g_{\mu\nu}V + \frac{i}{4}g_{\mu\nu}\Sigma \\ & - \frac{i}{4}(\bar{\Psi}_n\gamma_\mu\nabla_\nu\Psi_n - \nabla_\mu\bar{\Psi}_n\gamma_\nu\Psi_n \\ & + \bar{\Psi}_n\gamma_\nu\nabla_\mu\Psi_n - \nabla_\nu\bar{\Psi}_n\gamma_\mu\Psi_n)], \end{aligned} \quad (55)$$

with

$$\Sigma = e^\alpha{}_c(\bar{\Psi}_n\gamma^c\partial_\alpha\Psi_n - \partial_\alpha\bar{\Psi}_n\gamma^c\Psi_n). \quad (56)$$

Elements of Ricci tensor calculated from metric coefficients in Eq. (14) and the ansatz for Φ^m in Eq. (13) are substituted into Eq. (55), to give the set of equations

$$\begin{aligned} \frac{1}{2\delta^2}(\delta^2\alpha^2)_{,rr} - \frac{(\delta^2\alpha^2)_{,r}}{4\delta^2}\frac{\delta^2_{,r}}{\delta^2} + \frac{1}{r}\frac{(\delta^2\alpha^2)_{,r}}{\delta^2} = \\ \kappa[-V - \frac{i}{4}\Sigma + \frac{i}{2\delta\alpha}(\bar{\Psi}_n\gamma^0\partial_t\Psi_n - \partial_t\bar{\Psi}_n\gamma^0\Psi_n)], \end{aligned} \quad (57)$$

$$\begin{aligned} -\frac{1}{2\delta^2}(\delta^2\alpha^2)_{,rr} + \frac{(\delta^2\alpha^2)_{,r}}{4\delta^2}\frac{\delta^2_{,r}}{\delta^2} - \frac{1}{r}\alpha^2_{,r} = \\ \kappa[\alpha^2(F_{,r})^2 + V + \frac{i}{4}\Sigma \\ - \frac{i\alpha}{2}(\bar{\Psi}_n\gamma^1\partial_r\Psi_n - \partial_r\bar{\Psi}_n\gamma^1\Psi_n)], \end{aligned} \quad (58)$$

$$\begin{aligned} \frac{1}{r^2}(1 - \alpha^2) - \frac{1}{r}\alpha^2_{,r} - \frac{\alpha^2}{2r}\frac{\delta^2_{,r}}{\delta^2} = \\ \kappa[\frac{F^2}{r^2} + V + \frac{i}{4}\Sigma \\ - \frac{i}{2r}(\bar{\Psi}_n\gamma^2\partial_\theta\Psi_n - \partial_\theta\bar{\Psi}_n\gamma^2\Psi_n)], \end{aligned} \quad (59)$$

$$\begin{aligned} \frac{1}{r^2}(1 - \alpha^2) - \frac{1}{r}\alpha^2_{,r} - \frac{\alpha^2}{2r}\frac{\delta^2_{,r}}{\delta^2} = \\ \kappa[\frac{F^2}{r^2} + V + \frac{i}{4}\Sigma \\ - \frac{i}{2r\sin\theta}(\bar{\Psi}_n\gamma^3\partial_\phi\Psi_n - \partial_\phi\bar{\Psi}_n\gamma^3\Psi_n)]. \end{aligned} \quad (60)$$

The first in above equations is the (t, t) -component of Eq. (55), the second (r, r) -component, the third (θ, θ) -component, and the last (ϕ, ϕ) -component, respectively.

(θ, θ) -component of the Einstein equation is same as (ϕ, ϕ) -component of Einstein equation if

$$\bar{\Psi}_n\gamma^2\partial_\theta\Psi_n - \partial_\theta\bar{\Psi}_n\gamma^2\Psi_n = \frac{1}{\sin\theta}(\bar{\Psi}_n\gamma^3\partial_\phi\Psi_n - \partial_\phi\bar{\Psi}_n\gamma^3\Psi_n). \quad (61)$$

Using $\vec{\rho} \otimes \vec{\sigma}$ representation for γ -matrices and the mathematical identities;

$$\begin{aligned} \sigma_3\partial_\theta\mathcal{Y}_k^m = & (\pm\sigma_3m\cot\theta \pm \sigma_1\frac{i}{2\sin\theta})\mathcal{Y}_k^m \\ & \pm\sigma_3e^{\mp i\phi}\sqrt{(j \pm m + 1)(j \mp m)}\mathcal{Y}_k^{m \pm 1} \end{aligned} \quad (62)$$

this condition becomes

$$\begin{aligned} \sum_{m,m'}\psi_n^{\dagger(km')}\rho_1\psi_n^{(km)} \times \\ \left[\mathcal{Y}_k^{\dagger m'}\{\pm\sigma_3(m-m')\cot\theta \pm \sigma_1\frac{i}{\sin\theta}\}\mathcal{Y}_k^m \right. \\ \left. \pm\mathcal{Y}_k^{\dagger m'}\sigma_3e^{\mp i\phi}\sqrt{(j \pm m + 1)(j \mp m)}\mathcal{Y}_k^{m \pm 1} \right. \\ \left. \mp\mathcal{Y}_k^{\dagger m' \pm 1}\sigma_3e^{\pm i\phi}\sqrt{(j \pm m' + 1)(j \mp m')}\mathcal{Y}_k^m \right] \\ = \sum_{m,m'}\psi_n^{\dagger(km')}\rho_1\psi_n^{(km)}(m+m')\mathcal{Y}_k^{\dagger m'}\sigma_1\frac{i}{\sin\theta}\mathcal{Y}_k^m. \end{aligned} \quad (63)$$

The condition (63) can be satisfied if

$$\begin{aligned} m &= m', \\ m + m' &= 2m = \pm 1, \\ j &= \frac{1}{2}, \end{aligned} \quad (64)$$

since

$$\mathcal{Y}_{k(j=\frac{1}{2})}^{\pm\frac{1}{2}\pm 1} = 0. \quad (65)$$

Moreover, explicit calculations show us that

$$\begin{aligned} (\mathcal{Y}_{k(j=\frac{1}{2})}^{\pm\frac{1}{2}})^\dagger \mathcal{Y}_{k(j=\frac{1}{2})}^{\pm\frac{1}{2}} &= \frac{1}{4\pi}, \\ (\mathcal{Y}_{k(j=\frac{1}{2})}^{\pm\frac{1}{2}})^\dagger \sigma_1 \mathcal{Y}_{k(j=\frac{1}{2})}^{\pm\frac{1}{2}} &= \pm \frac{\sin\theta}{4\pi} \frac{k}{|k|}. \end{aligned} \quad (66)$$

These make Eqs. (57)-(60) θ - and ϕ -independent, which is consistent with spherically symmetric metric in Eq. (14), when we consider only $(j = \frac{1}{2}, m = \frac{1}{2})$ or $(j = \frac{1}{2}, m = -\frac{1}{2})$ fermions.

For the case $(j = \frac{1}{2}, m = \frac{1}{2})$ or $(j = \frac{1}{2}, m = -\frac{1}{2})$ with $k = \pm 1$, neutrino wave function is

$$\Psi_n(t, r, \theta, \phi) = e^{-iE_n t} \psi_n^{(k, \pm\frac{1}{2})}(r) \mathcal{Y}_k^{\pm\frac{1}{2}}(\theta, \phi). \quad (67)$$

Let $\psi_n(r) \equiv \psi_n^{(k, \frac{1}{2})}(r)$ or $\psi_n^{(k, -\frac{1}{2})}(r)$ and

$$\psi_n(r) = \frac{1}{r}(\alpha^2\delta^2)^{-\frac{1}{4}} \begin{pmatrix} \varphi_n \\ \chi_n \end{pmatrix}. \quad (68)$$

In this case the term Σ in Eq. (56) reads

$$\Sigma = -\frac{i}{r^2\alpha^2\delta^2}q_0 + \frac{i}{r^2\delta}q_1 + \frac{i}{r^2\alpha\delta}q_k, \quad (69)$$

with

$$q_0 = \frac{2E_n}{4\pi}r^2\alpha\delta\psi_n^\dagger\psi_n = \frac{2E_n}{4\pi}(\varphi_n^*\varphi_n + \chi_n^*\chi_n), \quad (70)$$

$$\begin{aligned} q_1 &= \frac{-i}{4\pi}r^2\alpha\delta(\psi_n^\dagger\rho_3\partial_r\psi_n - \partial_r\psi_n^\dagger\rho_3\psi_n) \\ &= \frac{-i}{4\pi}(\varphi_n^*\partial_r\varphi_n - \partial_r\varphi_n^*\varphi_n \\ &\quad - \chi_n^*\partial_r\chi_n + \partial_r\chi_n^*\chi_n), \end{aligned} \quad (71)$$

$$q_k = \frac{1}{2\pi}r\alpha\delta\frac{k}{|k|}\psi_n^\dagger\rho_1\psi_n = \frac{1}{2\pi r}\frac{k}{|k|}(\chi_n^*\varphi + \varphi_n^*\chi_n), \quad (72)$$

$$q_2 = \frac{1}{4\pi}r^2\alpha\delta\psi_n^\dagger\rho_2\psi_n = \frac{i}{4\pi}(\chi_n^*\varphi - \varphi_n^*\chi_n), \quad (73)$$

Using the last equations Einstein's equations are given by

$$\begin{aligned} \frac{1}{2\delta^2}(\delta^2\alpha^2)_{,rr} - \frac{(\delta^2\alpha^2)_{,r}}{4\delta^2}\frac{\delta^2_{,r}}{\delta^2} + \frac{1}{r}\frac{(\delta^2\alpha^2)_{,r}}{\delta^2} \\ = \kappa[-V + \frac{1}{4r^2}(\frac{q_0}{\delta^2\alpha^2} + \frac{q_1}{\delta} + \frac{q_k}{\alpha\delta})], \end{aligned} \quad (74)$$

$$\frac{\alpha^2}{r}\frac{\delta^2_{,r}}{\delta^2} = \kappa[\alpha^2(F_{,r})^2 + \frac{1}{2r^2}(\frac{q_0}{\delta^2\alpha^2} + \frac{q_1}{\delta})], \quad (75)$$

$$\frac{1}{r^2}(1 - \alpha^2) - \frac{1}{r}\alpha^2_{,r} - \frac{\alpha^2}{2r}\frac{\delta^2_{,r}}{\delta^2} = \kappa[\frac{F^2}{r^2} + V + \frac{1}{4r^2}(\frac{q_0}{\delta^2\alpha^2} - \frac{q_1}{\delta})], \quad (76)$$

and scalar field equation

$$\begin{aligned} \alpha^2 F_{,rr} + \frac{\alpha^2}{2}F_{,r}(\frac{\delta^2_{,r}}{\delta^2} + 2\frac{\alpha^2_{,r}}{\alpha^2} + \frac{4}{r}) - \frac{2F}{r^2} \\ - \frac{\partial V}{\partial F} + \frac{2g_y}{v}\frac{q_2}{r^2\delta\alpha}F = 0. \end{aligned} \quad (77)$$

IV. ASYMPTOTIC SOLUTIONS FOR LARGE r

As the global monopole solution [1], one component $\alpha(r)$ of the metric in Eq. (14), far away from the galactic core, can be asymptotically taken as

$$\alpha^2 \simeq \alpha_o^2 \equiv c - \frac{2M}{r}, \quad (78)$$

with a constant c when

$$F \simeq \eta, \quad V(\eta^2) \simeq 0, \quad \frac{\partial V}{\partial F}(\eta^2) \simeq 0. \quad (79)$$

Analysing Eqs. (74)-(77) with the assumption that q_i 's in Eqs. (70)-(73) are $\mathcal{O}(r^0)$, we can get an asymptotic solution for another component of the metric in Eq. (14) as

$$\delta^2 \simeq A^2 \ln\left(\frac{r}{2M}\right), \quad (80)$$

with a constant A . In this limit with the help of Eq. (68) the neutrino equation (53) becomes

$$\partial_r^2 \varphi_n \simeq -q^2(r)\varphi_n, \quad (81)$$

with

$$q(r) \equiv \sqrt{\frac{E_n^2}{\delta^2\alpha_o^4} - \frac{m_{o,n}^2}{\alpha_o^2}} \quad (82)$$

and $m_{o,n} = m_n - \frac{g_y}{v}\eta^2$.

In the region

$$0 < \ln\left(\frac{r}{2M}\right) \leq \frac{E_n^2}{A^2\alpha_o^2 m_{o,n}^2} \quad (83)$$

we have an asymptotic solution to Eq. (81),

$$\begin{aligned} \varphi_n &\simeq A_n e^{iq(r)r} + B_n e^{-iq(r)r}, \\ \chi_n &\simeq \frac{i}{m_{o,n}}[A_n(\frac{E_n}{\alpha_o\delta} - \alpha_o q(r))e^{+iq(r)r} \\ &\quad + B_n(\frac{E_n}{\alpha_o\delta} + \alpha_o q(r))e^{-iq(r)r}]. \end{aligned} \quad (84)$$

With above asymptotic solutions to Dirac's equation for neutrinos, the q_i in Eqs. (70)-(73) are given by

$$\begin{aligned} q_0 &\simeq \frac{E_n}{\pi}[B_n^2(1 + \frac{1}{m_{o,n}^2}(\frac{E_n^2}{\alpha_o^2\delta^2} + \alpha_o^2 q^2(r))) \\ &\quad + A_n B_n(1 + m_{o,n}^2)\cos(2qr)], \\ q_1 &\simeq \frac{2E_n}{\pi}B_n^2\frac{q^2}{\delta}, \\ q_k &\simeq \frac{2k}{\pi r|k|}A_n B_n\frac{\alpha_o q}{m_{o,n}}\sin(2qr), \\ q_2 &\simeq \frac{E_n}{\pi m_{o,n}\alpha_o\delta}[A_n B_n\cos(2qr) + B_n^2], \end{aligned}$$

with real constants A_n and B_n such that $|A_n| = |B_n|$. The q_i 's are $\mathcal{O}(r^0)$ or less and so are consistent with the assumption of Eq. (80). For $r > r_o$ with $\ln(\frac{r_o}{2M}) \equiv \frac{E_n^2}{A^2\alpha_o^2 m_{o,n}^2}$, $q(r)$ in Eq. (82) is pure imaginary and then wave functions of neutrinos become to be multiplied by exponentially decaying factors. Therefore we get *neutrino clustering* with the radius r_o . Taking $r_o \simeq 10^{23} \text{cm}$ which corresponds to the radius of our Galaxy and assuming that there exists a supermassive black hole at the center of Galaxy with a mass $M_{SBH} \simeq 3 \times 10^6 M_\odot$ [12], we can estimate the energy of neutrinos at $E_n \simeq 5 m_{o,n}$, where we have used the relations, $\frac{E_n^2}{A^2\alpha_o^2 m_{o,n}^2} \simeq \ln(\frac{r_o}{2M_{SBH}})$ and $A^2\alpha_o^2 \simeq \mathcal{O}(1)$.

V. ASYMPTOTIC SOLUTIONS FOR SMALL r

Next let us study the small r behaviors of components of the metric in Eq. (14), α and δ . For small r near the Galactic core we adopt the Thomas-Fermi (TF) approximation for the energy density and the pressure of neutrinos, as in the case of fermion stars [9][10][11]. From the local conservation law, $0 = (\bar{\Psi}\gamma^\mu\Psi)_{;\mu} = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\bar{\Psi}\gamma^\mu\Psi)$, we have the conserved, total number of neutrinos,

$$N = \int d^3x \sqrt{-g} \bar{\Psi}\gamma^t\Psi = \int d^3x \sqrt{-g} \Psi^\dagger\Psi e^t{}_0. \quad (85)$$

The number density of neutrinos in the TF approximation is given by

$$\langle\Psi^\dagger\Psi\rangle_{TF} = \frac{2}{(2\pi)^3} \int d^3q n_q = \frac{q_F^3(r)}{3\pi^2}, \quad (86)$$

where the Fermi distribution $n_q = \theta(q_F - q)$ with the fermi momentum q_F . From Eq. (86) and the time-independent Dirac's equation, $i\partial_t\Psi_n = E_n\Psi_n$, we can put

$$\begin{aligned} & \langle \frac{i}{2\delta\alpha} (\bar{\Psi}_n\gamma^0\partial_t\Psi_n - \partial_t\bar{\Psi}_n\gamma^0\Psi_n) \rangle_{TF} \\ &= \frac{2}{(2\pi)^3} \int d^3q n_q E_n(q) e^t{}_0 \equiv \rho(r), \end{aligned} \quad (87)$$

where ρ is the energy density of neutrinos and $e^t{}_0 = \frac{1}{\delta\alpha}$. The stress tensor of neutrinos has the following diagonal elements in the vierbein basis:

$$\begin{aligned} & \langle \frac{\alpha}{2i} (\bar{\Psi}_n\gamma^1\partial_r\Psi_n - \partial_r\bar{\Psi}_n\gamma^1\Psi_n) \rangle_{TF} = p_1, \\ & \langle \frac{1}{2ir} (\bar{\Psi}_n\gamma^2\partial_\theta\Psi_n - \partial_\theta\bar{\Psi}_n\gamma^2\Psi_n) \rangle_{TF} = p_2, \end{aligned} \quad (88)$$

and

$$\langle \frac{1}{2ir \sin\theta} (\bar{\Psi}_n\gamma^3\partial_\phi\Psi_n - \partial_\phi\bar{\Psi}_n\gamma^3\Psi_n) \rangle_{TF} = p_3, \quad (89)$$

respectively.

We assume, in the spherically symmetric spacetime we consider, that $p_1 = p_2 = p_3 \equiv p(r)$. From Dirac's equation (36) and its Hermitian conjugate, we have the relation;

$$\begin{aligned} & \frac{i}{\delta\alpha} (\bar{\Psi}_n\gamma^0\partial_t\Psi_n - \partial_t\bar{\Psi}_n\gamma^0\Psi_n) \\ & + i\alpha (\bar{\Psi}_n\gamma^1\partial_r\Psi_n - \partial_r\bar{\Psi}_n\gamma^1\Psi_n) \\ & + \frac{i}{r} (\bar{\Psi}_n\gamma^2\partial_\theta\Psi_n - \partial_\theta\bar{\Psi}_n\gamma^2\Psi_n) \\ & + \frac{i}{r \sin\theta} (\bar{\Psi}_n\gamma^3\partial_\phi\Psi_n - \partial_\phi\bar{\Psi}_n\gamma^3\Psi_n) = 2m_{\text{eff}}(r) \bar{\Psi}_n\Psi_n, \end{aligned} \quad (90)$$

with $m_{\text{eff}}(r) = m_n - \frac{gv}{v}F^2(r)$. Since the left hand side of above equation is same as $i\Sigma$ in Eq. (56), we have the following relation in the TF approximation.

$$\frac{i}{2}\langle\Sigma\rangle_{TF} \equiv \rho - 3p = m_{\text{eff}} \frac{v}{2g_y} Q, \quad (91)$$

where we have defined $\frac{v}{2g_y}Q(r) \equiv \langle\bar{\Psi}_n\Psi_n\rangle_{TF}$. Thus in the TF approximation the Einstein equations (57)-(60) read

$$\begin{aligned} & \frac{1}{2\delta^2}(\delta^2\alpha^2)_{,rr} - \frac{(\delta^2\alpha^2)_{,r}}{4\delta^2} \frac{\delta^2_{,r}}{\delta^2} + \frac{1}{r} \frac{(\delta^2\alpha^2)_{,r}}{\delta^2} \\ &= \kappa[-V + \frac{1}{2}\rho + \frac{3}{2}p], \end{aligned} \quad (92)$$

$$\begin{aligned} & -\frac{1}{2\delta^2}(\delta^2\alpha^2)_{,rr} + \frac{(\delta^2\alpha^2)_{,r}}{4\delta^2} \frac{\delta^2_{,r}}{\delta^2} - \frac{1}{r}\alpha^2_{,r} \\ &= \kappa[\alpha^2(F_{,r})^2 + V + \frac{1}{2}\rho - \frac{1}{2}p], \end{aligned} \quad (93)$$

$$\begin{aligned} & \frac{1}{r^2}(1 - \alpha^2) - \frac{1}{r}\alpha^2_{,r} - \frac{\alpha^2}{2r} \frac{\delta^2_{,r}}{\delta^2} \\ &= \kappa[\frac{F^2}{r^2} + V + \frac{1}{2}\rho - \frac{1}{2}p], \end{aligned} \quad (94)$$

which are consistent with those in the case of fermion stars [9]. The scalar field equation (21) reads

$$\alpha^2 F_{,rr} + \frac{\alpha^2}{2} F_{,r} (\frac{\delta^2_{,r}}{\delta^2} + 2\frac{\alpha^2_{,r}}{\alpha^2} + \frac{4}{r}) - \frac{2F}{r^2} - \frac{\partial V}{\partial F} + QF = 0. \quad (95)$$

If we assume that there exist regular solutions of the metric components, scalar fields, and the energy density and pressure of neutrinos to above equations (92)-(95) for small r , take series expansions as

$$\begin{aligned} \alpha^2 &= \sum_{n \geq 0} a_n r^n, \quad \delta^2 = \sum_{n \geq 0} b_n r^n \\ F &= \sum_{n \geq 0} \eta_n r^n, \quad \rho = \sum_{n \geq 0} \rho_n r^n, \quad p = \sum_{n \geq 0} p_n r^n, \end{aligned} \quad (96)$$

and substitute these expansions into Eqs. (92)-(95), then we get the following values for the coefficients.

$$\begin{aligned} a_0 &= 1, \quad a_1 = 0, \quad a_2 = -\kappa[\frac{1}{3}\rho_0 + \frac{\lambda}{12}\eta^4 + \frac{1}{2}\eta_1^2], \\ b_1 &= 0, \quad b_2 = b_0\kappa[\frac{1}{2}\rho_0 + \frac{1}{2}p_0 + \frac{1}{2}\eta_1^2], \\ \eta_0 &= 0, \quad \eta_2 = 0, \quad m_n \frac{v}{2g_y} Q_0 = \rho_0 - 3p_0 \dots \end{aligned} \quad (97)$$

We thus have asymptotic solutions as

$$\begin{aligned} F &= \eta_1 r + \mathcal{O}(r^3), \\ \alpha^2 &= 1 - \kappa[\frac{1}{3}\rho_0 + \frac{\lambda}{12}\eta^4 + \frac{1}{2}\eta_1^2]r^2 + \mathcal{O}(r^3), \\ \delta^2\alpha^2 &= \{1 + \kappa[\frac{1}{6}\rho_0 + \frac{1}{2}p_0 - \frac{\lambda}{12}\eta^4]r^2\} + \mathcal{O}(r^3), \end{aligned} \quad (98)$$

where we have reparametrized the time as $\sqrt{b_0}t \rightarrow t$. These solutions are consistent with the results in fermionic stars with a global monopole [11], even if they

[11] take $F \simeq 0$ for small r . The value of η_1 is determined by the matching condition that $F(r)$ should be continuous at $r = r_c$, which is the upper bound for small r . This gives us the relation,

$$\eta_1 r_c = \eta. \quad (99)$$

We might put $r_c \simeq 2M_{SBH}$ where M_{SBH} is the mass of the supermassive black hole at the center of our Galaxy [12].

VI. NEUTRINO CLUSTERING

In Z-burst models to explain the continuation of the cosmic ray spectrum beyond the GZK cutoff, Weiler [6] and Fargion [13] assumed that the relic neutrinos cluster in galaxies a few times the normal relic density, and found that the required flux of cosmic ray neutrinos is larger than previously suggested. Blanco-Pillado *et al.* [14] suggested that the relic neutrino density in the clustering might be $10^{12} - 10^{14}$ times the mean relic density, $n_M \simeq 54 \text{ cm}^{-3}$. Independently in Ref. [5] a neutrino cloud was considered as a sphere with no diffuse boundary such that the relic neutrino density is given by $n_\nu(r) = n_R \theta(R - r) + n_M \theta(r - R)$, where $n_R \simeq 10^{12} - 10^{16} \text{ cm}^{-3}$ and $R \simeq 10^{14} - 10^{20} \text{ cm}$. This can moderate the required incident flux of ultra high energy neutrinos.

From the results in the previous sections IV and V, we can have a cluster of neutrinos with a diffuse boundary such that the density of the relic neutrinos is

$$n_\nu(r) = n_c \theta(r_c - r) + n_o(r) \theta(r - r_c) \theta(r_o - r), \quad (100)$$

where n_c is the constant neutrino density for small r ($\leq r_c \sim 2M_{SBH}$) as in the section V, and $n_o(r) = n_c \frac{r_c^2}{r^2}$ is the neutrino density for $r_c < r \leq r_o$ as in the section IV. Here $r_o \simeq 10^{23} \text{ cm}$ (the radius of our Galaxy), $r_c \simeq 2M_{SBH} \simeq 10^{12} \text{ cm}$ (the Schwarzschild radius of a supermassive black hole at the center of the Galaxy), and $n_c \geq 10^3 (\frac{m_{eff}}{1 \text{ eV}})^3 \text{ cm}^{-3}$ [15].

VII. SUMMARY AND DISCUSSIONS

In the most general static spacetime with spherical symmetry, we explicitly show that only $j = 1/2$ fermions

can satisfy Einstein's equation, in the $\vec{\rho} \otimes \vec{\sigma}$ representation of γ -matrices [7]. It is also possible to show the fact in other representations of γ -matrices. Considering neutrinos coupled to $O(3)$ scalar fields $\vec{\Phi}$ via $\vec{\Phi}^2 \bar{\Psi} \Psi$ -interaction term, we had a global monopole solution [1] of scalar fields and asymptotic solutions of the metric components to Einstein's equation in Eqs. (78) and (80), for large r . The asymptotic solution of one metric component δ^2 can be given in series of more general functions as $\delta^2(r) \simeq \sum_n f_n \frac{r^n}{2M} \ln^n(\frac{r}{2M})$ with $f_n(r \rightarrow \infty) = \text{constant}$, which do not change the asymptotic behaviors of other solutions in this limit.

We adopted the Thomas-Fermi(TF) approximation for small r expansions of the energy density, pressure of neutrinos and so on. Since only s -waves are considered in the TF approximation, small r expansions in the section V are consistently related to asymptotic solutions obtained for large r in the section IV. More rigorous connection between two asymptotic solutions can be made by numerical methods. With the simple assumption, Eq. (100), about the radial dependence of the relic density of neutrinos clustered in the galactic halo, further studies of Z-burst models shall be possible. For small r solutions in the section V to have astronomical meaning, the global $O(3)$ symmetry breaking scale η and the scalar self-coupling constant λ in Eq. (12) should be very small, which might be realized in some Majoron models [17]. In such a case the deficit angle $1 - c \equiv \kappa \eta^2$ in Eq. (78) is negligible [1].

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